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# Heating of the intergalactic medium due to structure formation

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**ABSTRACT**
We estimate the heating of the intergalactic medium due to shocks arising from structure formation. Heating of the gas outside the collapsed regions, with small overdensities ($\frac{n_b}{\bar{n}_b} \ll 200$) is considered here, with the aid of Zel'dovich approximation. We estimate the equation of state of this gas, relating the density with its temperature, and its evolution in time, considering the shock heating due to one-$\sigma$ density peaks as being the most dominant. We also estimate the mass fraction of gas above a given temperature as a function of redshift. We find that the baryon fraction above $10^6$ K at $z = 0$ is $\sim 10\%$. We estimate the integrated Sunyaev-Zel'dovich distortion from this gas at present epoch to be of order $10^{-6}$.

**Key words:** cosmology: theory— large-scale structure of universe—intergalactic medium —galaxy:formation

## 1 INTRODUCTION

It has become evident from recent numerical simulations that a significant fraction of the baryons in the universe reside in the warm-hot phase of the intergalactic medium (WHIM), with temperatures of order $10^5$–$10^7$ K (Cen & Ostriker 1999, hereafter CO99). The gas in this phase is raised to high temperature by shock heating due to formation of structure. Recent simulations by Davé et al. (2000) and Croft et al. (2000) have also calculated that the equation of state of this phase is approximately $\rho \propto T$.

There have been a few analytical attempts too to understand the heating process in the intergalactic medium through analytic means. Pen (1999) pointed out that there is a need for non-gravitational heating in the intra-cluster and intergalactic medium (hereafter IGM) to avoid the constraints from the soft-X-ray background. Extra heating decreases the amount of clustering of the gas and therefore reduces the flux of soft X-ray radiation. Wu, Fabian & Nulsen (1999) have also addressed the question with detail calculations and concluded the same.

These are, however, relevant for heating of the gas which are already within collapsed objects. For example, the work of Wu et al (1999) refers to the heating of the gas which is already within a collapsed halo, with overdensities larger than $\sim 200$. The numerical simulations of CO99 and Davé et al. (1999), on the other hand, points out the heating in the gas which have overdensities much smaller than this.

It is interesting to note that Zel'dovich and his colleagues had reached similar conclusions to that of the recent numerical simulations in the context of their study of formation of pancakes. As a byproduct of their study of the formation of large pancakes, they had worked out the magnitude of gravitational heating of the intergalactic gas. Although much of the earlier motivation has been lost now, a substantial part of their work sounds prescient. To quote from Sunyaev and Zeldovich (1972)—"*It is possible that a significant fraction of the intergalactic gas (10-50 %) was not subjected to compression in the 'pancakes' and was heated only by the damped shock waves moving away from them.*" This is exactly what the numerical simulations have unearthed, namely, the heating of the gas which with overdensities smaller than $\sim 200$, outside the collapsed region but worked upon by shockwaves due to gravitational collapse.

In this *Letter*, we attempt to understand the heating of this phase of IGM with the help of Zel'dovich approximation. Since the gas in warm-hot IGM is only mildly non-linear, this approximation can shed light on the gravitational heating process, if used within its limitations. Below, we attempt to estimate the amount of the gravitational heating, and the sate of the gas, by including other heating and cooling effects. We also attempt to estimate the mass fraction of baryons which are affected by this heating as a function of redshift.

We assume a cosmological model with $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ and $h = 0.65$, with $\Omega_B\, h^2 = 0.015$, the big bang nucleosynthesis value.

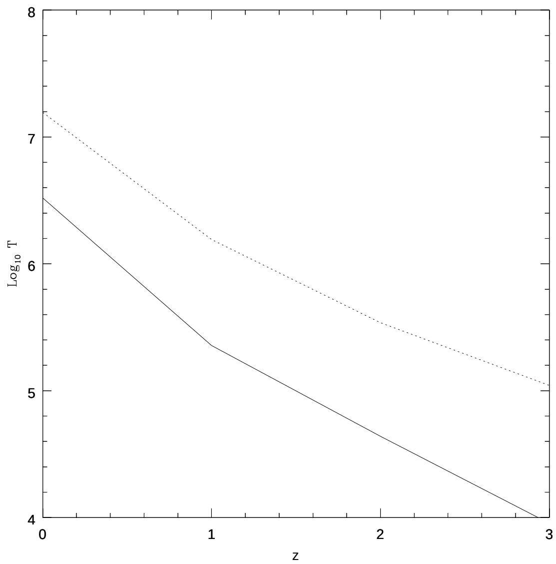


**Figure 1.** The maximum temperature from shocks due to structure formation is shown as a function of redshift (solid line). The dotted line is that from CO99.

## 2 SHOCK HEATING IN THE VICINITY OF COLLAPSED OBJECTS

Consider the gas surrounding a high density peak. As the gas flows inwards, it is compressed, and depending on its adiabatic exponent, it stops at a place away from the centre of accretion, and a shock wave travels outward. We will concentrate on this shockwave as it compresses and heats the very outer parts of the collapsed region. Sunyaev & Zel'dovich (1972; hereafter SZ72) tried to model this shock wave in the context of one-dimensional collapse of gas onto pancakes. In their idealized picture, as the gas flows towards the inner region, a singularity appears and a shock travels outwards through the gas. This shock velocity can be easily determined for the perturbation of a given lengthscale $\lambda(=2\pi/k)$, assuming a single sinusoidal perturbation. Suppose the singularity appears at a redshift $z_c$. They defined a parameter, $\mu$ which corresponds to a given Lagrangian coordinate, and is given by $\frac{\sin\pi\mu}{\pi\mu}=\frac{1+z}{1+z_c}$. The parameter $\mu$, therefore, is equivalent to a time parameter. In the case of a sinusoidal perturbation, it also gives the fraction of matter that has passed through the shock wave up to a given moment.

The velocity of matter falling onto the shock, $V_s$, as derived by SZ72, can be generalized for any cosmological model as,

$$\begin{aligned} V_s &\sim \frac{dz}{dt}\frac{\lambda}{2\pi}\frac{1}{(1+z_c)^2}(\mu\pi)^{1/2}\sin^{1/2}(\mu\pi) \\ &\sim \frac{dz}{dt}\frac{\lambda}{2\pi}\frac{1}{(1+z_c)^2}(\mu\pi)\,, \end{aligned} \tag{1}$$

where $\lambda$ is the comoving lengthscale of perturbation. The temperature behind the shock wave is given by $T_s \sim V_s^2 m_p/(6k_B)$, where $m_p$ is the mass of a proton and $k_B$ is the Boltzmann's constant (SZ72, eqn(2)).

Note that this temperature reaches a maximum at $\mu\sim 0.5$, when approximately half of the matter has passed through the shock. This happens when $1+z\sim(2/\pi)(1+z_c)$. The maximum temperature is given by, (noting that $\frac{dz}{dt}=H(z)\,(1+z)$)

$$\begin{aligned} T_{max} &\sim \frac{m_p}{6k_B}H(z)^2\Big(\frac{\lambda}{2\pi}\Big)^2(\mu\pi)^2\Big(\frac{1+z}{1+z_c}\Big)^2\frac{1}{(1+z_c)^2} \\ &\sim \frac{m_p}{6k_B}H(z)^2\frac{L_{ln}^2}{(1+z_c)^2} \end{aligned} \tag{2}$$

where we have written $L_{ln}=1/k$ for the comoving lengthscale of the perturbation, in the notation of CO99. This is the typical length of perturbations that goes non-linear at $1+z_c$. It is interesting to compare the Eqn (4) CO99 with this equation. They derived a value of $K=0.3$ from their simulation where the maximum temperature or, equivalently, the maximum sound velocity was given by $C_s^2=KH^2(L_{ln}/(1+z_c))^2$. Comparing with the above expression, we obtain, $K\sim 5/18$ for a monoatomic gas.

There is, however, a crucial difference. The parameter $L_{ln}$ in CO99 is defined as the perturbation that goes non-linear at a given redshift z. This provided the maximum temperature reached by the gas at a given z. In the above formulation, however, there are three important epochs: $z_{ln}$ is the epoch when the perturbation has an overdensity larger than unity and it goes non-linear, $z_c$ is the epoch when the singularity appears and $z_m$ is the epoch when $\mu\sim 0.5$, when the maximum gas temperature is achieved. Naturally $z_{ln}>z_c>z_m$. Here we have assumed that $z_{ln}\sim z_c$ but that it is larger than $z_m$. This difference becomes non-negligible especially at high redshifts. This difference is shown in Figure 1, where the maximum temperature reached at a given redshift is plotted for the $\Lambda$ cosmological model (as in CO99). The solid line is the prediction from the above formulation (with $z_{ln}\sim z_c>z_m$ and the dotted line is from CO99. The only difference is that we have treated $L_{ln}$ as the lengthscale of the perturbation that goes non-linear at a redshift slightly larger than $z_m$ to give the maximum temperature at $z_m$. Naturally, there is a difference in these two epochs, of order of a Hubble time.

## 3 EVOLUTION OF TEMPERATURE

In order to calculate the evolution of temperature of this gas, we will need to take into account all sources of heating and cooling and their rates. Firstly, let us consider the rate of heating due to these shocks. We should note here that the gas with different initial density (or overdensity) will go through the shockwave at different point of time. Gas with larger initial density will be closer to the singularity and will pass through the shock wave sooner. In the ideal case, the gas density profile is smooth and shock travels through it equally affecting all parts of it. In reality, however, we expect the gas closer to the collapsed region will be shocked and heated more effectively than the gas farther away.

We note that the average density profile of the gas away from the collapsed region is expected to be of the type $\rho\propto r^{-2/3}$, where $r$ is the perpendicular distance from the pancake or filament (Zel'dovich 1970). To treat the differential effectiveness of shock heating analytically, we shall define a timescale for the passing of the shockwave through the gas as,

$$t_s \sim \frac{\lambda}{1+z_c}\frac{1}{U}\Big(\frac{\rho}{\bar\rho}\Big)^{-3/2} \tag{3}$$

where $\bar{\rho}$ is the average ambient density, and where $U = V_s/3$ is the velocity of the shock front relative to the plane of symmetry (whereas $V_s$ is the velocity of matter impinging on the shock (e.g., Jones, Palmer & Wyse 1981)). The extra factor $\left(\frac{\rho}{\bar{\rho}}\right)^{-3/2}$ then accounts for the fact that gas with larger overdensity is heated more effectively than that with lower overdensity. This prescription is valid only for collapsing gas with different overdensities in a given perturbation. In this work, we attempt to calculate the equation of state of gas in a given 1-$\sigma$ perturbation. We then estimate the equation of state of gas in the IGM in general using the relevant filling factor of such perturbations.

We then write the shock heating rate as simply $T_{shock}/t_{shock}$. For a universe with $\Omega_\Lambda + \Omega_0 = 1$, one has,

$$\frac{dT_{shock}}{dz} \sim -0.4 \times 10^6 \left(\frac{\lambda/2\pi}{1\,\mathrm{Mpc}}\right)^2 (n_b/\bar{n_b})^{3/2} \times \mu^3 \frac{h^2(1+z)^2}{(1+z_c)^5}(\Omega_\Lambda + (1+z)^3\Omega_0) \quad (4)$$

We should, however, remember the maximum temperature that the gas can be raised to by the shocks, as discussed in §2. We therefore put an upper limit on the temperature, as given by eqn(2). This will reflect the physical fact that although lower density gas is not shocked as effectively as the higher density gas closer to the filament or pancake, higher density gas is not heated to indefinitely higher temperatures this way.

As was pointed out in SZ72, a useful approximation for $\mu$ is $\mu \sim \frac{1}{\pi}(6(1-\frac{1+z}{1+z_c}))^{1/2}$. We have used this approximation in our calculations below.

Before discussing other sources of heating, we should note here that this formulation is adequate only for a limited duration. Although in principle, the gas infall continues until $\mu = 1$, in reality, the approximations used to calculate $\mu$ breaks down for large values of $\mu$. We therefore consider the evolution of the temperature only until $\mu = 0.5$.

The second heating source, which is the adiabatic compression of the gas, is easily described as, (for $n_b/\bar{n_b} \gg 1$)

$$\frac{dT_{ad}}{dz} = \frac{2}{3}\frac{T}{(n_b/\bar{n_b})}\frac{d(n_b/\bar{n_b})}{dz} \quad (5)$$

We characterize the growth of the overdensity by the following equation,

$$\frac{d(n_b/\bar{n_b})}{dz} = -\eta\frac{(n_b/\bar{n_b})}{1+z}\,, \quad (6)$$

where $\eta$ equals unity for the linear regime in a $\Omega = 1$ universe. In the quasi-linear regime, $\eta$ could be large. For example, the overdensity evolves as $\delta \propto (1+z)^{-2.15}$ for the range of the scales where the power spectrum has $n = -2$ (Peacock 1999). We adopt a value of $\eta = 2$. The final result is found not to depend on its value strongly.

One cooling process is due to the expansion of the universe, and is given by,

$$\frac{dT_{ex}}{dz} = \frac{2T}{1+z}\,. \quad (7)$$

Cooling due to free-free radiation is given by,

$$\frac{dT_{ff}}{dz} = 0.22\Omega_b h\frac{(n_b/\bar{n_b})T^{1/2}(1+z_c)^2}{(\Lambda_0 + (1+z)^3\Omega_0)^{1/2}}\,. \quad (8)$$

Although cooling due to inverse Compton scattering becomes important at high redshift, at $z = 4$, the cooling time for Compton cooling ($\sim 9\times10^{12}(1+z)^{-4}$ yr $= 1.5\times10^{10}$ yr) is larger than that for free-free cooling for a gas at $10^6$ K and with an overdensity of $\sim 100$ ($\sim 10^9$ yr). It is shown below that at high redshifts ($z_c \gtrsim 3.5$) shock heating contributes to the equation of state only for gas with large overdensities, of order $\sim 100$. Since Compton cooling is not as efficient as Compton cooling for this gas, we neglect it in our calculation.

Combining all the heating and cooling processes, one has for the evolution of the gas temperature,

$$\frac{dT}{dz} = \frac{dT_s}{dz} + \frac{dT_{ad}}{dz} + \frac{dT_{ex}}{dz} + \frac{dT_{ff}}{dz} \quad (9)$$

We present the numerical solution of this equation below.

## 4 RESULTS

The process of heating due to structure formation is essentially statistical in nature. To track it analytically, however, we focus on the 1-$\sigma$ fluctuations. These are the perturbations that dominate the heating at a given redshift, as found in CO99. Fluctuations with higher degree of non-linearity at a given epoch would involve gas with very large overdensities, and does not concern us here. As has been emphasized earlier, here we are concerned with gas which is rather on the outskirts of highly non-linear perturbations.

To calculate the equation of state of gas at a given epoch, we therefore find out the relevant lengthscale, given the power spectrum which is COBE normalized. For the physical state of the gas at a redshift $z$, we determine the perturbation which goes non-linear at $z_c$, so that after evolving by a time period equivalent to $\mu = 0.5$, we reach the epoch $z$. In other words, to find the state of the gas at $z = 0(1,2)$, we adopt $z_c = (1+z)*(\pi/2) - 1 \sim .5(2.,3.5)$. We find the scale of the perturbations which are one-$\sigma$ at this epoch, according to the relevant power spectrum. The value of $L_{ln}$ at $z_c \sim 0.5(2.,3.5)$ is $\sim 6(1.6,0.65)\,h^{-1}$ Mpc. The initial values of $\rho_b/\bar{\rho}$ is taken to be in the range of 1–50. The initial temperature has been fixed at $3\times10^4$ K, which is the temperature reached by the IGM gas through photoionization heating. The results for the state of gas at $z = 0,1,2$ are shown in Figure 2. The dotted line shows the equation of state as found by Davé et al (2000).

We can estimate the fraction of the mass that has a given temperature in the following way. First, we note that, in the formulation of SZ72, if the density of gas which has just entered the shock front is $\rho_i$, then the fraction of mass ($f$) that has already gone through the shockwave is given as, $(\rho_i/\bar{\rho}) \sim 3/(\pi^2 f^2)$. If $T$ is the temperature of the gas corresponding to the initial density $\rho_i$, then $f$ is the fraction of mass with temperature larger than $T$. To be precise, this approximation is valid for the case of instantaneous cooling, which is a reasonable assumption for $\mu \lesssim 0.1$ ($z_c \gtrsim 0.5$, as can be seen from eqns (4) & (9)). In other words, this approximation is valid only for small values of $f(\ll 1)$.

We multiply this fraction with the (comoving) number density of the one-$\sigma$ peak, as given in Bardeen et al. (1986), to derive the fraction of mass which has temperature larger than a given value. The results for the fractions at $z = 0,1,2$ are shown in Figure 3. Unfortunately, the limitations of the

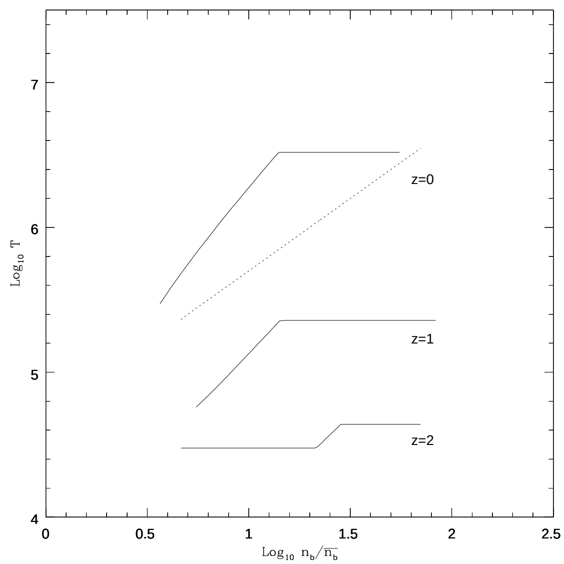


**Figure 2.** Temperature of the gas is plotted against its overdensity $n_b/\bar{n}_b$. The solid curves are for gas at $z = 0, 1, 2$ from top to bottom. The dotted line shows the equation of state found by Davé et al. (2000) (their Fig 6).

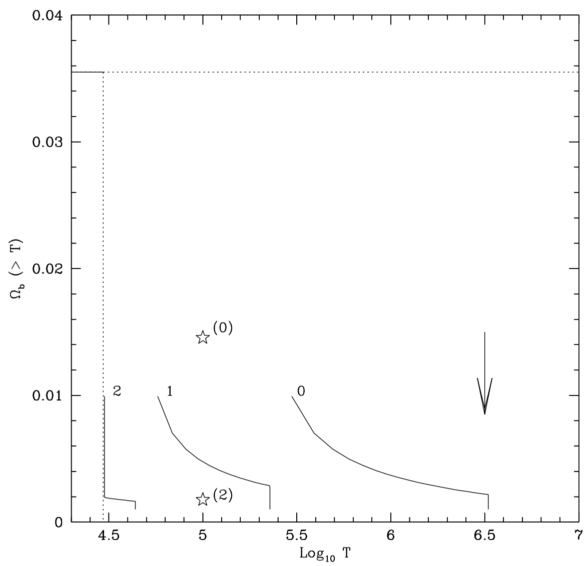


**Figure 3.** The fraction of shock heated diffuse gas above a given temperature $\Omega_B(> T)$ is plotted against logarithm of $T$. The three solid curves from right to left are for gas at $z = 0, 1, 2$ respectively. The stars are the mass fraction at $z = 0$, and $z = 2$ found by Croft et al. (2000, their §2). The horizontal dotted line refers to the total baryonic gas in the universe and the vertical dotted line shows the lower limit of the temperature due to photoionization. The arrow is from the limit of soft X-ray emission for gas with $n_b/\bar{n_b} \sim 100$ at $z = 0$.

single sinusoidal wave approximation do not allow us to draw a full curve, as one has to stop at $f \sim 1$ (in reality, the above approximation is valid only for small values of $f$). The curves, however, can serve as pointers to what one-$\sigma$ peaks can do to the diffuse IGM.

We also show by stars the mass fraction derived by Croft et al. (2000) at $z = 0, 2$, and the fractions are 41% and 5% respectively. To compare these numbers with the curves in Figure 3, we should remember that the curves show the result of heating by one-$\sigma$ density peaks only. In reality, there will be contribution from higher sigma peaks, taking the gas to higher temperatures at rarer places. Although we do not have the fraction for gas above $10^5$ K at $z = 0$, a naive extrapolation of the existing curve above $10^6$ K is consistent with this value. We note here that the curve for $z = 0$ shows that the fraction of gas above $10^6$ K is of order 10%.

We note here that the mass fractions from the simulation of CO99 are much larger than these. Since the simulations use different techniques and employ different resolutions, it is not obvious to what do these discrepancies owe their existence and if they are of much importance.

## 5 DISCUSSIONS

The equation of state for low density gas in Figure 2 depends on the particular form of the timescale for passing of the shockwave in eqn (3). as is evident from the analytical solution (eqn (10)). In reality, modelling the efficiency of shock heating might add additional parameters and the resulting equation would therefore have some scatter.

In Figure 3, we also plot the limits from the observations of the soft X-ray background. After the subtraction of the discrete sources, it now appears that a flux of 4 keV cm$^{-2}$ s$^{-1}$ sr$^{-1}$ keV$^{-1}$ at 0.25 keV can be taken as an upper limit to any possible contribution from diffuse matter in the IGM (e.g., Wu, Fabian & Nulsen 1999). We apply this limit to our result for IGM at $z = 0$. If the gas at temperature $T$ (with the corresponding density $n(T)$, as given in Fig 2 for $z = 0$) has a filling factor $\epsilon_T$, then the flux at 0.25 keV is proportional to $Flux\,(keV/cm^2\,s\,sr\,keV) \propto \epsilon_T n(T)^2\,T^{-0.5}\,\exp(-0.25\,keV/k_BT)\,(c/H_0)$ and this allows us to calculate $\epsilon_T$ as a function of $T$. Since $\epsilon_T$ is found to decrease very quickly with $T$, we can approximate $\epsilon(> T) \sim \epsilon_T$ and use it to put a limit on the mass fraction of gas with temperature larger than a given value $T$. This limit is shown as a dashed arrow in Figure 3 for gas with $n_b/\bar{n_b} = 100$ at $T = T_{max}$ for $z = 0$. The limit for lower density gas is less restrictive. The above limit is obviously uncertain to the extent that the appropriate path length differs from the approximate $c/H_0$.

In this calculation we have used a metallicity of 0.01 solar abundance, which is the metallicity found in the Lyman-$\alpha$ absorption systems at high redshift, and is relevant for the diffuse gas considered here. It is possible that the metallicity of this diffuse gas increases in time, as found in the numerical simulation by Cen & Ostriker (1999b). It is, however, not yet known from observations how this metallicity evolves in time, and its dependence, if any, with the density and clumpiness of the gas. Any metallicity will make the above limit more stringent; in other words, make the contribution of the diffuse shock-heated IGM gas towards to the soft X-ray background much more prominent. As the detail numerical simulation of the soft X-ray background radiation by Croft et al. (2000) shows, the contribution of the (enriched) shocked heated diffuse gas is comparable to

the upper limit of the unresolved X-ray emission in the soft band.

Given the relation between density and temperature, we can estimate the resulting Sunyaev-Zel'dovich distortion on the cosmic microwave background. Defining $f_{los}$ to be the fraction of the line of sight going through hot filamentary structures, one can estimate the integrated Compton $y$ parameter aas,

$$y \sim 10^{-6} \left(\frac{n_b/\bar{n_b}}{10^2}\right) \left(\frac{T_e}{10^6\,K}\right) \left(\frac{c/H_0}{10^3\,Mpc}\right) \left(\frac{f_{los}}{0.03}\right). \qquad (10)$$

The (comoving) volume fraction of 1-$\sigma$ peaks from Bardeen et al. (1986) is of order 3%. Another way of estimating this is to use the fact that the gas with density $\gtrsim n_b$ occupies a lengthscale $\frac{\lambda}{(1+z_c)}(n_b/\bar{n}_b)^{-3/2}$, The Compton $y$-parameter is approximated as,

$$y \sim \frac{2\lambda}{(1+z_c)}(n_b/\bar{n}_b)^{-3/2}\,\frac{n_b k_B T_e}{m_e c^2}\,\sigma_T\,, \qquad (11)$$

where $\sigma_T$ is the Thomson cross-section. For the case at $z = 0$, $y$ for a single structure is found to be of order $\sim 4 \times 10^{-8}$ for $n_b/\bar{n}_b \sim 100$. There will be, however, of order $((c/H_0)/L_{ln}) \times f_{los} \sim 30(f_{los}/0.03)$ such structures in one line of sight, and so the total distortion will amount to $y \sim 10^{-6}$.

## 6 SUMMARY

We have applied the Zel'dovich approximation to estimate the heating of the diffuse intergalactic medium by shocks associated with 1-$\sigma$ density peaks in structure formation at different redshifts. We are able to reproduce the equation of state of the warm-hot IGM found in recent numerical simulations. We estimate the baryon fraction of the gas above $10^6$ K at the present epoch to be at least $\sim 0.1\Omega_b$. The integrated Sunyaev-Zel'dovich distortion from the diffuse IGM filaments amounts to $y \sim 10^{-6}$.

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